K L UNIVERSITY

Pre-Ph.D. Examination, Mathematics Paper – III

Theory of Semigroups

Syllabus

Unit-I : Functions on a semigroup

Semigroup, special subsets of a semigroup, special elements of a semigroup, relation and functions on a semigroup, Transformations, Free semigroups.

Unit-II : Ideals and Related concepts

Subdirect products, Completing prime ideals and Filters, Completely semiprime ideals, Semilattices of simple semigroups, Weekly commutative semigroups, separative semigroups, π - semigroups.

Unit-III : Ideal Extensions

Extensions and Translations, Extensions of a Weekly Reductive semigroup, strict and pure extensions, Retract Extensions, Dense extensions, Extensions of an Arbitrary semigroups, Semilattice compositions.

Unit-IV : Completely Regular semigroups

Completely regular, completely simple semigroups , semilattices of Rectangular groups, strong semilattice of completely simple semigroups, subdirect product of a semilattice and a completely simple semigroup.

Unit- V : Inverse Semigroups

The natural partial order of an inverse semigroup, partial right congruences on an inverse semigroup, Representations by one-to-one partial transformations, Homomorphisms of inverse semigroups, semilattices of inverse semigroups.

Note : 1. 8 Questions to be set out of which 5 Questions to be answered.
2. Questions should be uniformly distributed from all the units.

Prescribed text Book :

1. Introduction to Semigroups by **Mario Petrich; Charles E. Merrill** Publishing Company.
2. The algebraic theory of semigroups volume II, **By A.H.Clifford and G.B.Preston** **A**merican mathematical society.

Reference Text Book :

1. The Algebraic Theory of Semigroups by **A.H.Clifford and G.B.Preston;** American Mathematical Society, First edition.

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SEMIGROUPS

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer ANY FIVE from the following.

1) For any element a of a semigroup S, show that i) $L(a) = a \cup Sa$ ii) $R(a) = a \cup aS$ *iii)* $J(a) = a \cup aS \cup Sa$

2) If φ is a homomorphism of a semigroup S into a semigroup T, then the relation ρ on S defined by $a \rho b$ ifand only if $a\varphi = b\varphi$, is a congruence on S , and $S/\rho\ S\varphi$. Conversely, if ρ is a congruence on
S , then the mapping $a \to a\rho$ is a homomorphism of S onto S/ρ.

3) Show that every semigroup is a subdirect produc t of subdirectly irreducible semigroups.

4) Let S be a semigroup , **I** be a semiprime idel and M be an m-system of S such that I $M = \varphi$ and let M* be any m-system of S maximal relative to the properties : MM^*, I M^* = Then show that $S \backslash M^*$ is a minimal prime ideal of s containing **I**

5) A semigroup S is a retract of every extension if and only if S has an identity.

6) Show that the following conditions on a semigroup S are equivalent.

i) S is completely simple

ii) S is completely regular and simple

iii) S is regular and all its idempotents are primitive.

iv) S is regular and weakly cancellative.

v) S is regular and for any $a, x \in S, a = axa$ implies $x = xax$

7) If H be an inverse subsemigroup of the inverse semigroup S. Then show that HW is a closed inverse subsemigroup of S.

8) Show that an effective representation of an inverse semigroup S is the sum of a uniquely determined family of transitive effective representations of S.