K L UNIVERSITY

Pre- Ph.D. Examination, Mathematics Paper – II

Topology

Syllabus **Max Marks:100**

Unit – I Topological Spaces and Continuous Functions

Topological spaces, basis for a topology, the order topology, the product topology on X x Y, the sub space topology, closed sets and limit points, continuous functions, the product topology, the metric topology.

Unit – II Connectedness and compactness

Connected spaces, connected subspaces of the real line, compact spaces, compact subspaces of the real line, limit point compactness.

Unit – III Countability and separation axioms

The countability axioms, the separation axioms, normal spaces, the urysohn lemma, the urysohn metrization theorem.

Unit – IV The Tychnoff Theorem

The Tychnoff Theorem, Completely Regular Spaces, The Stone – Cech Compactification.

Unit – V Complete metric spaces and function spacess

Complete metric spaces, compactness in metric spaces, pointwise and compact convergence, ascoli's theorem.

Note : 1. 8 Questions to be set out of which 5 Questions to be answered.

2. Questions should be uniformly distributed from all the units.

Prescribed text Book:

1. Topology by **James Dugundji**; Universal Book Stall, New Delhi.
2. Introduction to Topology by **G.F.Simmons**; Tata McGraw-Hill Publishing Company.

Reference Text Book:

1. Topology by **James R.Munkres**; Prentice-Hall, Second edition.

Model Question Paper, Mathematics paper - II

Topology

Attempt any five questions from the following

5x20 = 100

1. (a) Describe the lower limit topology T in the set R of real numbers. Is T finer than the usual topology on R? Justify.

(b) Suppose B and B' are base for topologies T and T' on a set X. If every is a subset of some? Justify.

2. (a) Describe the dictionary topology on R X R and prove that this topology coincides with the product topology where is R equipped with the discrete topology and the second factor R has the usual topology.

(b) Show that for a subset A of X, $\bar{A}$ = A U A^{I}

3. (a) Show that the Cartesian product of connected spaces is connected.

(b) Give an example of a connected space which is not path connected.

4. (a) Show that a metrizable space X is compact if and only if X is sequentially compact.

(b) Show that the Cantor set is compact.

5. Which of the following are true? Justify

(i) If X and Y are second countable so is X X Y

(ii) If X and Y are Lindelof spaces so is X X Y

6. (a) Show that every regular space with a countable basis is normal.

(b) Show that a connected normal space having more than one element is normal.

7. (a) S.T A metric space X is complete iff every Cauchy's sequence in X has a convergent sequence.

(b) If X is a complete topological space ,Show that the space C(X,R) of all continuous real valued functions on X is complete under the metric C defined by C(f,g)= $\mathop{\text{lub}}_{x \in X}$ {|f(x)-g(x)|}

8. S.T a metric space X is compact iff X is complete and totally bounded.