KL UNIVERSITY: GUNTUR
DEPARTMENT OF MATHEMATICS
Pre Ph.D. Examinations

Paper II: Timescale calculus

Eight questions are to be set and the student has to answer five in three hours of duration:

UNIT-1 **Basic definitions**: Jump operators, left and right dense, left and right scattered, Induction principle. Differentiation, Properties Leibnitz rule, examples and applications.

UNIT-2 **Integration**: Regulated function rd-continuous, Existence of pre-antiderivative and antiderivative, Mean value theorem, chain rule, Intermediate vale theorem and L'Hopitals rule.

Unit-3 : **First order linear equations:** Hilger's complex plane, the exponential function, examples of exponential functions, The regressive linear dynamic equations, initial value problems and variation of constants formula..

Unit-4 : **Second order linear equations**: Wronskians, Linear operator, Abel's theorem, Hyperbolic and Trigonometric functions, Method of factoring, reduction of order, Euler-Cauchy equations, variation of parameters formula.

Text Books: Dynamic equations on time scales, an introduction with Applications. Martin Bohner and Allan Peterson, BirKhauser, Boston.

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DEPARTMENT OF MATHEMATICS

Pre-Ph.D Degree Examination

MODEL PAPER

Paper II: Timescale calculus

Time: 3Hours **Max.Marks:100**

Answer any FIVE questions from following, each question carries equal marks.

1. Define ,classify and illustrate the following with examples:
   a) Left and right dense,
   b) Left and right scattered
   c) Graininess function.
2. Assume that $f : T \to R$ is a function and let $f : T \in T^k$. Then prove the following:
   (i) If f is differentiable at t, then f is continuous at t.
   (ii) If f is continuous at t and t is right scattered, then fis differentiable at t with $f^{\Delta}(t) = \frac{f(\sigma(t)) - f(t)}{\mu(t)}$.
   (iii) If t is right-dense, then fis differentiable at t iff the limit $\lim_{s \to t} \frac{f(t) - f(s)}{t - s}$.exists as a finite number.
   (iv) If f is differentiable at t, then $f(\sigma(t)) = f(t) + \mu(t) f^{\Delta}(t)$.
3. a) Find the first derivative t^2 on an arbitrary time scale.
   b) Define regulated and rd-continuous functions with examples.
4. Assume $g : \Re \to \Re$ is continuous and $g : T \to \Re$ is delta differentiable on T^k, and $f : \Re \to \Re$ is continuously differentiable. Then prove that there exists c in the interval [t,σ(t)] with $(f \circ g)^{\Delta}(t) = f'(g(c))g^{\Delta}(t)$.
5. Suppose $y^{\Delta}(t) = p(t)y$ is regressive and $t_0 \in T$. Then prove that $e_p(t,t_0)$ is a solution of the initial value problem $y^{\Delta}(t) = p(t)y$, $y(t_0)=1$ on time scale T.
6. Apply the variation constants formula, solve the following initial value problems
   (i) $y^{\Delta} = 2y + t, \quad y(0) = 0, \quad where \quad T = \Re$
   (ii) $y^{\Delta} = 2y + 3, \quad y(0) = 0, \quad where \quad T = Z$
7. Solve the dynamic equation $y^{\Delta\Delta} - (t+3)y^{\Delta} + 3ty = 0$ on the time scale T=N.
8. Determine the solution of Euler-Cauchy dynamic equation $t\sigma(t)y^{\Delta\Delta} - 4ty^{\Delta} + 6y = 0$ on a general time scale $T \subset (0, \infty)$.

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