Special Functions Syllabus

UNIT-I: The Gamma and Beta Functions

The Gamma function ,A series for $\Gamma'(z)/\Gamma(z)$,Evaluation of $\Gamma'(1)$, the Euler product for $\Gamma(z)$,the difference equation $\Gamma(z+1) = z\Gamma(z)$,evaluation of certain infinite products , Euler 's integral for $\Gamma(z)$, the Beta function , the value of $\Gamma(z)\,\Gamma(1-z)$, the factorial function , Legendre 's duplication formula , Gauss multiplication theorem , a summation formula due to Euler .

UNIT-II: BESSEL FUNCTIONS

Definition of J_n (x) , Bessel's differential equation , Differential recurrence relation, A pure recurrence relation, A generating function, Bessel's integral, Index half an odd integral, modified Bessel function, orthogonality property for J_n (x).

UNIT-III: LEGENDRE'S POLYNOMIALS

Definition of P_n (x) , Differential recurrence relations, the pure recurrence relation, Legendre's differential equation, the Rodrigue's formula , orthogonality property , special properties of P_n (x), more generating functions, Laplace's first Integral form , Expansion of x^n

UNIT-IV: HERMITE POLYNOMIALS

Definition of H_n (x),Recurrence relations , the Rodrigue's formula ,other generating functions ,integrals, the Hermite polynomials as ${}_2F_0$, orthogonality , expansion of polynomial s, more generating functions.

UNIT-V: LAGUERRE POLYNOMIALS

The Laguerre polynomial definition , generating functions, , recurrence relations, the Rodrigue's formula, the differential equation , orthogonality, expansion of polynomials , special properties ,other generating functions , the simple Laguerre polynomials.

TEXT BOOK:

(1) Special functions by E.D. Rainville, MacMillan company, New York, 1960.

Model Question Paper

Time:3 hours **Max Marks:100**

Note: Answer ANY FIVE from the following.

1 (a) Find the relation between the beta and the gamma function.

(b) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

2 (a) State and prove the Legendre's duplication formula.

(b) Evaluate $\int_0^{\infty} e^{-ax} x^{m-1} \sin bx\, dx$ **, by using gamma function.**

3 (a) Prove the orthogonality property for the Bessel function.

(b) Show that $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$.

4 (a) State and prove the Rodrigue's formula for Legendre polynomials.

(b) Express the polynomial $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ **in terms of Legendre polynomials.**

5 (a) State and prove the generating function for Hermite polynomials.

(b) Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.

6 (a) Show that $\int_0^{\infty} e^{-x} L_m(x) L_n(x) dx = 0,\ m \neq n$.

(b) Evaluate $\int_0^{\infty} e^{-2x} [L_3(2x)]^2 dx$.

7 (a) Using Rodrigue's formula, show that $P_n(x)$ **satisfies the differential equation**

$$\frac{d}{dx}\left[(1+x)^2 \frac{d}{dx}[P_n(x)]\right] + n(n+1)P_n(x) = 0.$$

(b) Prove that $\int_0^{\infty} e^{-ax} J_0(bx) dx = \frac{1}{\sqrt{a^2+b^2}}$.

8 (a) Evaluate $\int_{-\infty}^{\infty} e^{-x^2} [H_2(x)]^2 dx$.

(b) Prove that $L_n'(x) = L_{n-1}'(x) - L_{n-1}(x)$.