

KL UNIVERSITY: GUNTUR
DEPARTMENT OF MATHEMATICS
Pre Ph.D. Examinations
Paper III: Dynamical systems on Time scales

Eight questions are to be set and the student has to answer five in three hours of duration:

UNIT-1 Self-Adjoint equations: Wronskian matrix, Lagrange Identity, Abel's formula, Hermitian, Riccati equation. Sturm's separation and Comparison theorems.

UNIT-2 Linear Systems and Higher order equations: Regressive matrices, Existence and Uniqueness theorem, matrix exponential function, Variation of constants, Liouville's Formula, Constant coefficients.

UNIT-3: Asymptotic behavior of solutions: Growth and dichotomy conditions, Levinson's perturbation Lemma properties and applications.

UNIT-4: Dynamic Inequalities: Gronwall's Inequality, Bernoulli's Inequality, Holders and Minkowski's inequalities. Lyapunov inequalities.

Text Book: Dynamic equations on time scales, an introduction with Applications. Martin Bohner and Allan Peterson, BirKhauser, Boston.

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Pre-Ph.D Degree Examination
MODEL QUESTION PAPER
Paper III: Dynamical systems on Time scales

Time: 3 Hours **Max.Marks:100**
Answer any FIVE questions from following, each question carries equal marks.

1. State and prove the comparison theorem for initial value problems.
2. Let $A \in \mathfrak{R}$ be an $n \times n$ matrix valued function on T and suppose that $f: T \rightarrow \mathfrak{R}^n$ is rd-continuous. Let $t_0 \in T$ and $y_0 \in \mathfrak{R}^n$. Then prove that the initial value problem $y^\Delta(t) = A(t)y + f(t), y(t_0) = y_0$ has a unique solution $y: T \rightarrow \mathfrak{R}^n$.
3. Derive Gronwall's Inequality.
4. Assume $f \in Crd$ and $x(t,s)$ be the Cauchy function for $L(x(t)) = (px^\Delta)^\Delta(t) + q(t)x^{\sigma(t)} = 0$ for all $t \in T$ then $x(t) = \int_a^t x(t,s)f(s)\Delta s$ is the solution of the initial value problem $Lx=f(t), x(a)=0, x^\Delta(a) = 0$.
5. Write the dynamic equation $x^{\Delta\Delta} - 5x^\Delta + 6x = 0$ on $T = \mathfrak{R}$ in self adjoint form. Also express the difference equation $x(t+2) - 5x(t+1) + 6x(t) = 0$ on $T = \mathbb{Z}$ in self adjoint form.
6. Let $A \in \mathfrak{R}$ be a 2×2 matrix valued function and assume that X is a solution of $X^\Delta = A(t)X(t)$, then prove that X satisfies Liouville's formula.
7. State and prove Lyapunov inequality.
8. Solve the vector dynamic equation $x^\Delta = \begin{pmatrix} 3 & 1 \\ -13 & -3 \end{pmatrix} x$ for any time scale T .

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