KL UNIVERISTY :: GUNTUR
DEPARTMENT OF MATHEMATICS
Pre-Ph.D. Examinations
Paper II : DIFFERENTIAL EQUATIONS

Eight questions are to be set and the student has to answer five in three hours of duration

UNIT-I : System of linear differential equations: system of first order equations, existence and uniqueness theorem, fundamental matrix, non- homogeneous linear system, linear systems with constant coefficients.

UNIT-II : Existence and Uniqueness of solutions: Introduction, preliminaries, successive approximations, Picard's theorem, continuation and dependence on initial condition, existence of solutions in the large interval.

(Scope and treatment as in Chapters : 4 and 5 of Text book (1))

UNIT- III : Oscillation theory and boundary value problems : Qualitative properties of solutions, the sturm comparison theorem, Eigen values, Eigen functions

UNIT- IV : Power series solutions and special functions : Series solutions of first order and second order linear differential equations, ordinary points, regular singular points. Gauss's hyper geometric equation, the point at infinity.

UNIT V : Non-linear equations : Autonomous systems, the phase plane and its phenomena, type of critical points, stability, critical points and stability for linear systems, stability by Liapunov's direct method, simple critical points of non linear systems.

(Scope and treatment as in Chapters : 4 ,5(sections 25-29) and 8 (sections 40-44)of Text book (2))

Text Books :

1. Text book of ordinary differential equations by S.G.Deo, V. Lakshmikantham and V. Raghavendra, second Edition, Tata McGraw - Hill publishing Company Ltd., New Delhi, 2002.
2. Differential equations with applications and historical Notes by George F. Simmons, Tata McGraw - Hill publishing Company Ltd., New Delhi, 1972.

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MODEL PAPER

Paper II : DIFFERENTIAL EQUATIONS

Time : 3 hours **Max. Marks : 100**

Answer any **Five** questions from following, each question carries equal marks.

1. (a) Find the fundamental system of solutions for the system of equations

$$x_1'(\mathrm{t}) = x_1(t), \quad x_2'(t) = 2x_2(t), \text{ for all } t \in [0,1].$$

   (b) Compute the solution of the following non-homogeneous system $x' = Ax + b(t)$, where

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}, b(t) = \begin{bmatrix} e^t \\ e^{-t} \end{bmatrix} \text{ and } x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

2. (a) Determine exp(At) for the system $x' = Ax$, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}.$$

   (b) Prove that if x(t) is the solution of the initial value problem $x' = f(t,x), x(t_0) = x_0$ on Some interval if and only if x(t) is solution of the corresponding integral equation.

3. (a) If f(t, x) is continuous function on $|t| \leq \infty$, $|x| < \infty$ and satisfies Lipschitz condition on the strip S_a for all a > 0, where Sa = $\{(t, x) : |t| \leq a, |x| < \infty\}$. Then show that the initial value problem $x' = f(t,x), x(t_0) = x_0$ has a unique solution existing for all t.

   (b) State and prove Picard's theorem.

4. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of

$$y^{11} + p(x)y^1 + q(x)y = 0$$

   then the zeros of these functions are distinct and occur alternately in the sense that $y_1(x)$ vanishes exactly once between any two successive zeros of $y_2(x)$, and conversely.

5. (a). Express $\sin^{-1} x$ in the form of a power series $\sum a_n x^n$ by solving $y^1 = (1-x)^{-1/2}$

   Use this result to obtain the formula

$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2}.\frac{1}{3}.\frac{1}{2^3} + \frac{1.3}{2.4}.\frac{1}{5.2^5} + \frac{1.3.5}{2.4.6}.\frac{1}{7.2^7} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(b). Find the general solution $(1+x^2)y^{11}+2xy^1-2y=0$ of in terms of power series in x.

Can you express this solution by means of elementary functions?

6. (a). Find the power series solution of $x^2y^{11}+(3x-1)y^1+y=0$

(b). Find two independent Frobenius solutions of each of the equations

$$x^2y^{11}-x^2y^1+(x^2-2)y=0$$

7. Describe the relation between the phase portraits of the systems

$$\begin{cases} \dfrac{dx}{dt}=F(x,y) \\ \dfrac{dy}{dt}=G(x,y) \end{cases} \quad \text{and} \quad \begin{cases} \dfrac{dx}{dt}=-F(x,y) \\ \dfrac{dy}{dt}=-G(x,y) \end{cases}$$

8. Sketch the phase portrait of the equations $\dfrac{d^2x}{dt^2}=2x^3$, and show that it has an unstable isolated critical point at the origin.