KL UNIVERSITY
M.Phil / PRE-Ph.D EXAMINATION
DEPARTMENT OF MATHEMATICS
SYALLABUS

PAPER-II: Distributions & Estimation Theory

Unit 1 : **DISTRIBUTIONS**

Discrete And Continuous Distributions (Binomial,Poisson,Geometric,Hyper Geometric ,Rectangular,Normal,Gamma Distributions and their Properties),Bi-Variate and Multivariate Normal Distributions, Exponential Family of Distributions

Unit 11: ***LIMIT THEOREMS***

Modes of convergence, Weak law of large numbers, Strong law of large numbers. Limiting moment generating functions, Central limit theorem.

Unit 111: **SAMPLE MOMENTS AND THEIR DISTRIBUTIONS**

Random sampling, sample characteristics and their distributions- x^2 ,t and F distributions distribution of ($\bar{X}$,S^2) in sampling from a normal population. Sampling from a Bi-variate normal distribution

Unit IV : ***THEORY OF POINT ESTIMATION***

Problem of point estimation, Properties of estimates, Unbiased estimation, Lower bound for variance of estimate, Rao- Blackwell theorem, Method of moments, Maximum likelihood estimates, Bayes & Minimax estimation, Minimal sufficient statistic

Unit V : ***CONFIDENCE INTERVAL ESTIMATION***

Shortest length confidence intervals, Relation between confidence estimation and hypothesis testing, unbiased confidence intervals, Bayes confidence intervals

PRESCRIBED BOOK

An introduction to Probability theory and Mathematical Statistics-V.K. Rohatgi, Wiley Eastern Publications first edition- 1975) [Chapters 5,6,7,8,11]

Additional Reading: Introduction to Mathematical Statistics (Fourth edition) Robert Hogg & Allen Craig

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MODEL QUESTION PAPER
PAPER-II - Distribution and Estimation Theory

Time: 3 Hours **Max.Marks: 100**

Answer any FIVE questions.

All Questions carries equal marks.

1. a) Obtain the mean and variance of a truncated Binomial distribution truncated at X=0.
   b) Derive the p.d.f. of Poisson distribution truncated at the origin and find its mean and variance.
2. a) Let X_1, X_2 be independent random variable with Xi follows b(n_i,$\frac{1}{2}$), i=1,2. What is the PMF for X_1- X_2+ n_2?
   b) Let X and Y be independent geometric RVs. Show that min(X,Y) and X-Yare independent.
3. a) Let $X_n \xrightarrow{p} X$, and g be continuous function defined on R. Then $g(X_n) \xrightarrow{p} g(X)$ as n →∞.
   b) State and prove Borel-Cantelli Lemma.
4. a) Let (X_1, Y_1), (X_1, Y_1), . . . , (X_1, Y_1) be a sample from a bi-variate population with variances $\sigma_1^2,\ \sigma_2^2$ and covariance $\rho\sigma_1\sigma_2$. Then E$ES_1^2 = \sigma_1^2,\ ES_2^2 = \sigma_2^2\ and\ ES_{11} = \sigma_1\sigma_2$
   b) Let X_1, X_2, . . . , X_n be a random sample from N(μ, σ^2). Compute the first four sample moments of $\bar{X}$ about the origin and abut the mean Also compute the fit four sample moments of S^2 about the mean.
5. a) Derive the characteristic function of a chi-square distribution. Establish its reproductive property.
   b). Let X and Y be independent normal RVs. A sample of n=11 observations on (X,Y) produces sample correlation coefficient r=.40.Find the probability of obtaining a value of R that exceeds the observed value.
6. a) Find the general form of the distribution of X such that a random sample X_1, X_2, . . . , X_n from the distribution as sufficient statistic.
   b) State and prove Rao-Blackwell theorem. State Lehmann and Schaffe theorem. Explain its use.
7. a) Explain moments method of estimation. Under regularity t_y conditions to be stated by you, prove that M.L estimator is asymptotically efficient.
   b) Let X_1, X_2, . . . , X_n be a sample from G(1,θ). Find the shortest-length confidence interval for θ at level (1-α), based on a sufficient statistic for θ.
8. a) Let X_1, X_2, . . . , X_n be i.i.d. with PDF $f_\theta(x) = \frac{\theta}{x^2}\ , x \geq \theta, and = 0\ otherise$. Find the shortest length (1-α)-level unbiased confidence interval for θ based on the pivot $\frac{\theta}{X_{(1)}}$.
   b) Let X_1, X_2, . . . , X_n be a sample from U(0, θ). Show that the unbiased confidence interval for θ based on the pivot matrix$\frac{X_i}{\theta}$, coincides with the shortest length confidence interval based on the same pivot.