KL UNIVERSITY
Pre-Ph.D. Examination
BOUNDARY VALUE PROBLEMS
Syllabs

UNIT-1 : **System of linear differential equations:** system of first order equations, existence and uniqueness theorem, fundamental matrix, non-homogeneous linear systems, linear systems with constant coefficients.

Unit-2 : **Existence and Uniqueness of Solutions:** introduction, preliminaries, successive approximations, Picard's theorem, continuation and dependence on initial condition, existence of solutions in the large interval.

(Scope and treatment as in chapters: 4 and 5 of Text book (1))

Unit-3: **Nonlinear boundary value problems:** Kinds of boundary value problems associated with Non-linear second order differential equations, generalized Lipschitz condition, failure of existence and uniqueness of linear boundary value problems, simple nonlinear BVP, standard results concerning initial value problems.

Unit-4: **Relation between the first and second boundary value problems:** relation between uniqueness intervals, relation between existence intervals.

Unit-5: **Contraction mapping:** introduction, Contraction mappings, boundary value problems, a more generalized Lipschitz condition.

(Scope and treatment as in chapters: 1, 2 and sections 3.1 to 3.4 of chapter 3 of Text book (2))

Text Books:

1. Text book of ordinary differential equations by S. G. Deo, V. Lakshmikantham and V. Raghavendra, Second edition, Tata McGraw-Hill Publishing Company Ltd, New Delhi (2002).
2. Non-linear two point boundary value problem by P. B. Bailey, L. P. Shampine and P. E. Waltman, Academic press, New York and London (1968).

KL UNIVERSITY
Pre-Ph.D. Examination
BOUNDARY VALUE PROBLEMS

MODEL QESTION PAPER

Time: 3Hours **Max.Marks:100**

Answer any FIVE questions from following, each question carries equal marks.

1. (a) Find the fundamental system of solutions for the system of equations

$$x_1'(\mathrm{t}) = x_1(t),\quad x_2'(t) = 2x_2(t),\ \text{for all } t \qquad ,1].$$

(b) Compute the solution of the following non-homogeneous system $x' = Ax + b(t)$, where

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}, b(t) = \begin{bmatrix} e^{t} \\ e^{-t} \end{bmatrix} \text{ and } x(0) = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$

2. (a) Determine exp(At) for the system $x' = Ax$, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}.$$

(b) Prove that if x(t) is the solution of the initial value problem $x' = f(t,x), x(t_0) = x_0$ on some interval if and only if x(t) is solution of the corresponding integral equation.

3. (a) If f(t, x) is continuous function on $|t| \le \infty$, $|x| < \infty$ and satisfies Lipschitz condition on the strip S_a for all a > 0, where Sa = {(t, x) : $|t| \le a$, $|x| < \infty$}. Then show that the initial value problem $x' = f(t,x), x(t_0) = x_0$ has a unique solution existing for all t.

(b) State and prove Picard's theorem.

4. Define the kinds of boundary value problems and discuss the solution of nonlinear boundary value problem $y''(t) + |y(t)| = 0$, satisfying $y(0) = 0, y(b) = B$.

5. (a) Let a < c < b. If uniqueness hold for all second boundary value problems $y''(t) + f(t, y(t), y'(t)) = 0$, $y(a) = A, y'(c_1) = m$, whenever $c_1 \in [a,c]$ and if uniqueness hold for all second boundary value problems $y''(t) + f(t, y(t), y'(t)) = 0$, $y'(c_1) = m, y(b) = B,$ whenever $c_1 \in [c,b]$, then show that uniqueness holds for all first boundary value problems $y''(t) + f(t, y(t), y'(t)) = 0$, $y(a) = A, y(b) = B.$

(b) Obtain the Green's function for second order equation $y''(t) + F(t, y(t), y'(t)) = 0,$ satisfying the zero boundary conditions $y(a) = 0, y(b) = 0$,
And also find bounds for Green's function.

6. (a) Let $f(t,y)$ be continuous on $[a,b] \times (-\infty,\infty)$ and satisfies $|f(t,x) - f(t,y)| \le K|x - y|$, then show that first boundary value problem $y'' + f(t,y) = 0,\ y(a) = A, y(b) = B$ has

unique solution whenever $\frac{K(b-a)^2}{\pi^2} < 1$.

(b) Discuss the failure of existence and uniqueness of the linear boundary value problem

$$y''(t) + y'(t) = 0, \quad y(0) = 0, \quad y(b) = B.$$

7. (a) Obtain the existence and unique solution of first boundary value problem

$$y''(t) + f(t, y(t), y'(t)) = 0, \; y(a) = A, y(b) = B$$

by matching the solutions of two initial value problems, provided $f(t, y, y')$ is continuous and satisfies Lipschitz condition.

(b) Let $f(t, y, y')$ be continuous on $[a,b] \times (-\infty, \infty) \times (-\infty, \infty)$ and satisfies

$$| f(t, y, y') - f(t, x, x') | \leq K | y - x | + L | y' - x' |.$$

Then show that the boundary value problem $y''(t) + f(t, y(t), y'(t)) = 0$, $y(a) = A, y(b) = B$ has one and only one solution provided

$$\frac{K(b-a)^2}{8} + \frac{L(b-a)}{2} < 1.$$

8. (a) Define generalize Lipschitz condition.

(b) Suppose $f(t, y, y')$ be continuous on $[a,b] \times (-\infty, \infty) \times (-\infty, \infty)$ and satisfies

$$| f(t, y, y') - f(t, x, x') | \leq p(t) | y - x | + q(t) | y' - x' |.$$

If the equation $u''(t) + q(t)u'(t) + p(t)u(t) = 0$ has a solution satisfying $u(t) = 0$, $u'(t) = m$ on [a,b]. Then show that the second boundary value problem $y''(t) + f(t, y(t), y'(t)) = 0, \; y(a) = A, y'(b) = m$ has one and only one solution